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CRT-710

INCREASES IN RISK AND THE
DEMAND FOR INSURANCE*

by

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- * We thank H. Schlesinger and P. Viala for their comments on a previous version and A. Mathieu, C. Laflamme and J. Lafontaine for their contributions in the preparation of the manuscript.

Centre de recherche sur les transports - Publication #710
Département de sciences économiques - Cahier 9021

October 1990

91-01573/

Dépôt légal - 4e trimestre 1990

ABSTRACT

Many people anticipate that a risk averse agent faced with an exogenous mean preserving increase in risk will demand more insurance. This widespread belief does not always turn out to be true in the insurance market and, more generally, in many other economic examples. This negative result has prompted many theoretical investigations in order to obtain intuitively acceptable results. Some authors have searched for conditions on the utility function, others have presented subclasses of mean preserving increases in risk, while a third group has considered the two kinds of restrictions jointly. It is interesting to notice that none of these contributions have applied their analysis to the insurance problem, although Meyer and Ormiston (1989) have interpreted the nature of insurance contracts as simple risk reducing transformations. The object of this article is to fill this gap in literature. By examining the study of the coinsurance coverage, we show that one of its specifications, namely the linearity of the payoff in the decision variable and in the random element, does not preclude the applicability of well known theorems to the demand for insurance.

Keywords : Mean preserving increases in risk, linear payoffs, demand for insurance, coinsurance, relatively strong increase in risk, strong increase in risk.

RÉSUMÉ

Plusieurs individus anticipent qu'un agent riscophobe qui subit un accroissement de risque à moyenne constante va demander plus d'assurance. Cette anticipation n'est pas toujours vérifiée dans l'exemple de l'assurance et, plus généralement, dans plusieurs applications économiques. Ce résultat négatif a engendré plusieurs investigations théoriques afin d'obtenir des résultats intuitivement acceptables. Certains auteurs ont donné des conditions sur la fonction d'utilité des agents, d'autres ont présenté des restrictions sur les accroissements de risque à moyenne constante alors qu'un troisième groupe a considéré les deux formes de restrictions simultanément. Il est intéressant de noter qu'aucune de ces contributions a appliqué son analyse aux contrats d'assurance même si Meyer et Ormiston (1989) ont interprété la nature des contrats d'assurance en termes de réduction simple de risque. L'objectif de cette contribution est de combler cette lacune dans la littérature. En se limitant au problème de la coassurance nous montrons qu'une de ses spécifications, la linéarité du payoff dans la variable de décision et dans l'élément aléatoire, n'empêche pas l'application des théorèmes connus à la demande d'assurance.

Mots clés : accroissement de risque à moyenne constante, payoff linéaire, demande d'assurance, coassurance, accroissement de risque relativement fort, accroissement de risque fort.

INTRODUCTION

Many people anticipate that a risk averse agent faced with an exogenous mean preserving increase in risk will demand more insurance to maintain an adequate level of protection.

This widespread belief does not always turn out to be true as already implicitly shown in a very general context by Rothschild and Stiglitz (1970, 1971) when they first presented the notion of a mean preserving increase in risk (M.P.I.R.) or of a mean preserving spread. For instance, when they applied their model to the simplest portfolio problem, they were unable to obtain the a priori intuitive result that an M.P.I.R. in the return of the risky asset will reduce its holdings by all risk averse individuals. In order to generate clear-cut comparative statics results they needed to put additional restrictions either on the shape of the utility function or on that of the original density. They arrived at the same conclusion for many other applications such as savings under uncertainty, the behaviour of the competitive firm under uncertainty and the firm production problem. This observation has prompted further investigations. Indeed, some authors have searched for conditions on the utility function — besides monotonicity and risk aversion — that would yield the desired result [Drèze and Modigliani (1972), Diamond and Stiglitz (1974), Dionne and Eeckhoudt (1987), Dionne, Eeckhoudt and Briys (1989)]. Another branch of the literature has tried to stay close to the assumption of risk aversion while presenting subclasses of M.P.I.R. that would produce the kind of comparative statics results that one expects [Meyer and Ormiston (1983, 1985), Black and Bulkley (1989), Eeckhoudt and Hansen (1980, 1983, 1984)]. Notice also that, in some papers, the two kinds of restrictions are jointly considered [Sandmo (1970, 1971), Meyer and Ormiston (1989)].

One may notice that none of the authors mentioned so far have applied their analysis to the insurance problem, although Meyer (1989) and Meyer and Ormiston (1989) have interpreted the nature of insurance contracts as simple risk reducing transformations.

The object of this article is to fill this gap in literature. By examining the study of the optimal coinsurance coverage, we find that one of its specifications, namely the linearity of the payoff in the decision variable and in the random element does not preclude the applicability of well known theorems to the demand for insurance. We begin the paper with a review of the main contributions in the field. We then introduce the characteristics of the demand for coinsurance and, in Section 3, we present our main result. A short conclusion proposes potential extensions.

1. MEAN PRESERVING INCREASES IN RISK: A REVIEW

The first distinction to be made in the field of M.P.I.R. concerns the difference between global and marginal increases in risk. For a global increase in risk, the economic environment moves from initial certainty to an uncertain situation. This class of problems has been extensively analyzed by Kraus (1979) and Katz (1981) for one-dimensional utilities and by Drèze and Modigliani (1972) for general utility functions in a two-period context. An important finding, in the one-dimensional environment on which we shall concentrate the discussion, is to present conditions under which all risk averse agents react in a way that is in agreement with intuition. Kraus and Katz derived their results for global increases in risk by considering the following problem:

$$\underset{\alpha}{\text{Max}} E [U(Z(\alpha, x))] \quad (1.1)$$

where x is a random variable which is degenerate in the initial situation ($x \equiv x_0$);

U is an increasing and concave utility function of Z ;

Z is a payoff function; and

α is a decision variable.

In all reviewed contributions, Z is assumed strictly concave in α . It is assumed that x is an advantageous outcome by taking Z_x to be strictly positive. This arbitrary definition is not at all restrictive. However, we shall keep it in this review in order to simplify the discussion.

Let us define α^* as the optimal level of the control variable in the initial (here, non-risky) situation.¹ The assumption that $Z_{\alpha\alpha}$ is always negative ensures that the second order condition for a maximum is satisfied. When the agent is shifted to uncertainty (for exogenous reasons) from x_0 to x with $E(x) = x_0$, α'' will denote his new optimal position which is obtained by solving (1.1). In order to give a sign to $(\alpha'' - \alpha^*)$, Kraus and Katz simply needed to focus on the shape of Z and did not have to restrict U beyond the standard assumptions that $U' > 0$ and $U'' < 0$.

When marginal increases in risk are considered, two risky situations are compared and equation (1.1) becomes the starting point. This can be written:

¹When the initial situation is non risky, (1.1) amounts to : $\underset{\alpha}{\text{Max}} U(Z(\alpha, x_0))$ where x_0 is the degenerate value of x .

$$\text{Max}_{\alpha} \int_{x_2}^{x_3} U(Z(\alpha, x)) dF(x) \quad (1.1')$$

where $F(x)$ is the original cumulative distribution function with support in the bounded interval $[x_2, x_3]$. $F(x)$ is assumed here to be continuous, but the following analysis could easily be accommodated for discrete random variables.

In the context of marginal changes in risk, the definition of an M.P.I.R., namely a mean preserving spread, was formulated by Rothschild and Stiglitz (1970). They presented three equivalent definitions of an M.P.I.R. One of them, called the integral condition, is written

$$\begin{aligned} (a) \quad & \int_{x_1}^{x_4} [G(x) - F(x)] dx = 0 \\ (b) \quad & \int_{x_1}^y [G(x) - F(x)] dx \geq 0 \text{ for all } y \in [x_1, x_4] \end{aligned} \quad (1.2)$$

where : $G(x)$ is the cumulative distribution function after the exogenous increase in risk;
 $[x_1, x_4]$ is a bounded interval that contains the support of $G(x)$ with $x_1 \leq x_2 \leq x_3 \leq x_4$.

Specifically, condition (b) indicates that G has more weight in the tails than F , while condition (a) guarantees the equality of the means. One of the major results of Rothschild and Stiglitz (1970) is that a mean preserving spread reduces expected utility under risk aversion, but does not always reduce the optimal level of the risky activity (α). This "negative" result has prompted much further research. We concentrate here on the articles whose main focus was the restrictions on the changes from F to G .

In fact, Rothschild and Stiglitz themselves already took a step in that direction by showing that in the comparison between bets satisfying conditions (a) and (b), intuitive comparative statics results emerge in the sense that M.P.I.R. reduce both expected utility and the level of the risky activity. Notice, however, that bets represent a very specific case since only two outcomes for x are allowed both in the initial and final situations.²

²For example, in the insurance problem that will be analyzed in detail in the next section, two sufficient restrictions to obtain the result that a risk averse individual will demand a greater insurance coverage when the distribution of losses becomes riskier (mean preserving spread) is either because U is quadratic or because the random variable can take only two values: no loss with probability $(1-p)$ and a positive loss with probability p .

This restriction was nicely removed by Meyer and Ormiston (1983, 1985) in two companion papers where they defined a "strong increase in risk". The basic idea behind the definition is the following: while Rothschild and Stiglitz redistribute probability mass located at a set of points to the right and to the left without restrictions, Meyer and Ormiston require that the shifts to the right and to the left have to be outside (or at the extreme points) of the initial interval of the distribution $F(x)$ [see Meyer and Ormiston (1985), p. 429 for more details]. Formally, a strong increase in risk is obtained by adding the following condition (c) to (a) and (b) in (1.2) :

$$\begin{aligned} (c) \quad & G(x) - F(x) \text{ is non increasing on } (x_2, x_3) \\ & \text{where support } F(x) \text{ is contained in } [x_2, x_3] \\ & \text{and support } G(x) \text{ is contained in } [x_1, x_4]. \end{aligned} \quad (1.3)$$

This additional condition means that the weight taken from $F(x)$ is never redistributed inside (x_2, x_3) but transferred either to x_2 and x_3 themselves or outside these limits. While the strong increase in risk is obviously a special case of Rothschild and Stiglitz's definition (because of the additional condition (c)) it results in a generalization of : 1) the notion of a global increase in risk; 2) the case of bets; and 3) the concept of a squeeze [Eeckhoudt and Hansen (1980, 1983)].

Despite its advantages, the restriction imposed by Meyer and Ormiston remains rather strong and it is legitimate to try to improve it. This goal was reached by Black and Bulkley (1989) who presented the notion of a "relatively strong increase in risk" to compare risky situations. In their approach, they relax the need to transfer weight from inside the original density to points that are outside or just at its end points. More precisely, they use the idea that an increase in risk should raise the likelihood of the extreme values of the initial density and this is the reason why they consider the behaviour of the likelihood ratio function $f(x)/g(x)$ where $f(x)$ and $g(x)$ are the probability density functions associated to F and G . Formally, the definition of a relatively strong increase in risk reads as follows³ :

³As pointed out by Black and Bulkley (1989), (a) and (b) in (1.4) are sufficient for $G(x)$ to represent a mean preserving spread in the Rothschild and Stiglitz sense. Moreover, as we shall see in Section 3, conditions (c) and (d) are not always required to obtain the desired results of comparative statics analysis.

$$(a) \int_{x_1}^{x_4} [G(x) - F(x)] dx = 0;$$

- (b) For all points in $[x_5, x_6]$, $f(x) \geq g(x)$
 and for all points outside this interval
 $f(x) \leq g(x)$ with $x_1 \leq x_2 \leq x_5 \leq x_6 \leq x_3 \leq x_4$,
 where $[x_1, x_4]$ is defined here as the support of $G(x)$ and
 $[x_2, x_3]$ as the support of $F(x)$;

$$(c) \frac{f(x)}{g(x)} \text{ is non-decreasing in the interval } [x_2, x_5]; \quad (1.4)$$

$$(d) \frac{f(x)}{g(x)} \text{ is non-increasing in the interval } (x_6, x_3].$$

Condition (b) means that the weight taken from inside the support of $F(x)$, that is $[x_2, x_3]$, can be sent either outside $[x_5, x_6]$ or, even at its extremities, inside $[x_1, x_4]$. Conditions (c) and (d) specify how the weights sent to the extremities of $F(x)$ have to be allocated. This discussion reveals that the relatively strong increase in risk includes, as a special case, a strong increase in risk. To give an example different from those presented by Black and Bulkley, consider Figure 1 in which two triangular densities are compared:

(Figure 1, here)

Obviously between x_2 and x_5 , $\frac{f(x)}{g(x)}$ is increasing and the reverse occurs between x_6

and x_3 . Notice that this example satisfies Black and Bulkley's definition without being a strong increase in risk.

After having presented the definition of both a strong increase and a relatively strong increase in risk, we now turn to a general presentation of the results obtained by using them. They can be synthesized in the following theorem:

THEOREM 1: If α' and α'' maximize $E[U(Z(\alpha, x))]$ under $F(x)$ and $G(x)$ respectively, sufficient conditions for $\alpha'' < \alpha'$ for all risk averse decision makers are such that:

- i) $G(x)$ represents either a strong or a relatively strong increase in risk;
- ii) $Z_x \geq 0$, $Z_{\alpha\alpha} \geq 0$, $Z_{\alpha\alpha\alpha} \leq 0$, $Z_{\alpha\alpha} < 0$.

PROOF:

See the corresponding articles.

Before introducing the insurance model, let us discuss briefly another type of risk increase often used for comparative statics analyses. First proposed by Sandmo (1970, 1971) as a "stretching" of a density around a constant mean, this notion of increasing risk was recently extended by Meyer (1989) and Meyer and Ormiston (1989) who showed that an initial random variable x can be transformed by using a single value function $t(x)$. The transformation $t(x)$ is defined as deterministic in order to distinguish it from other transformations presented above. Under particular conditions (a non-decreasing, continuous and differentiable function), the transformation $t(x)$ represents an M.P.I.R. of the random variable x if the function $k(x) \equiv t(x) - x$ is such that

$$\begin{aligned} \text{a) } & \int_{x_1}^{x_3} k(x) dF(x) = 0 \\ \text{b) } & \int_{x_1}^y k(x) dF(x) \leq 0 \text{ for all } y \in [x_2, x_3] \end{aligned} \tag{1.5}$$

As usual, condition (a) preserves the mean of the random variable, while condition (b) is equivalent to the second integral condition introduced by Rothschild and Stiglitz. To obtain intuitive comparative statics results, the transformation $t(x)$ must be restricted to the case of a simple transformation and conditions on U must be introduced. For instance, linear transformations such as the stretching of Sandmo (1970) or the squeeze of Eeckhoudt and Hansen (1980) are examples of simple deterministic transformations. In fact, Meyer and Ormiston (1989) have proved the following theorem:

THEOREM 2: If α^* and α^{**} maximize $EU(Z(\alpha, x))$ under x and $t(x) \equiv x + k(x)$ respectively, sufficient conditions to obtain $\alpha^{**} < \alpha^*$ for all risk averse decision makers are:

- a) $U(\cdot)$ corresponds to a decreasing (absolute) risk averse function;
- b) $Z_x \geq 0$, $Z_{xx} \leq 0$, $Z_{bx} \geq 0$, and $Z_{bxx} \leq 0$;
- c) $t'(x) \geq 0$ and $t(x)$ is a simple increase in risk.

PROOF:

See Meyer and Ormiston (1989).

It is interesting to observe that Theorem 2 contains conditions related to both the utility function and the transformation of the random variable. This suggests that even simple deterministic transformations of random variables represent quite general increases in risk. We are now ready to analyze the demand for insurance.

2. THE COINSURANCE MODEL

In this paper, we refer to the classical insurance problem (Mossin, 1968; Briys, Dionne, Eeckhoudt, 1989) in which the individual is endowed with a wealth, W_0 , and a risk of damage defined by a random variable x with support contained in the interval $[0, L]$, where L stands for the value of the physical asset submitted to the risk.

To protect himself against this damage, the individual has the opportunity to buy a coinsurance contract paying an indemnity αx ($0 \leq \alpha \leq 1$) for a fixed premium $P = \lambda \alpha \mu$, where λ is equal to unity plus the loading factor and $\mu \equiv E(x)$.

Given these specifications, the individual's optimization problem is

$$\text{Max}_{\alpha} E[U(W_0 - (1-\alpha)x - \lambda\mu\alpha)] \quad (2.1)$$

with first order condition

$$E[U'(Z)(x - \lambda\mu)] = 0 \quad (2.2)$$

where Z stands for the value of final wealth. Strict concavity of U guarantees that the second order condition for a maximum is satisfied. Notice also that the value of λ will always be such that the optimal α that solves (2.2) is an interior solution for all transformations of the distribution function.

At this stage, it is worth pointing out the differences between the present problem and those presented in the preceding section. A major distinction concerns the sign of $Z_{\alpha\alpha}$. In the two models that are the closest to our own problem [Meyer and Ormiston (1985), and Black and Bulkley (1989)], $Z_{\alpha\alpha}$ needs to be strictly negative in order to derive Theorem 1 (see especially p. 431 in Meyer and Ormiston and the definition of b^* in Black and Bulkley, p. 126). Of course, in our problem $Z_{\alpha\alpha}$ is equal to zero. However, as we shall see this difference will not preclude the applicability of both Black and Bulkley's and Meyer and Ormiston's conditions to obtain comparative statics results.⁴

Other differences are given in the table below:

Table 1
Comparison between conditions of Theorem 1
and characteristics of the insurance problem

Conditions of Theorem 1		Characteristics of the coinsurance problem
1	$Z_{\alpha\alpha} < 0$	$Z_{\alpha\alpha} = 0$
2	$Z_x \geq 0$	$Z_x < 0$
3	-----	$Z_{xx} = 0$
4	$Z_{\alpha x} \geq 0$	$Z_{\alpha x} = 1$
5	$Z_{\alpha xx} \leq 0$	$Z_{\alpha xx} = 0$

The second difference, which results from the fact that x is a damage, is unimportant since it only alters the type of proof. Condition (3) in the insurance problem will be useful in the proof of our main result while it seems unimportant in Theorem 1. The last two conditions illustrate other specific aspects of the insurance problem.

⁴See Dionne and Eeckhoudt (1990) for a more general discussion.

3. INCREASES IN RISK AND THE DEMAND FOR INSURANCE

PROPOSITION: If α' and α'' maximize $E[U(Z(x, \alpha))]$ under $F(x)$ and $G(x)$ respectively, sufficient conditions to obtain a greater insurance coverage ($\alpha'' > \alpha'$) for all risk averse decision makers are such that:

- i) $G(x)$ represents a relatively strong increase in risk;
- ii) $Z(x, \alpha)$ is linear in α and x .

PROOF:

The proof starts from the first order condition corresponding to the initial density $f(x)$

$$E[U'(Z) \cdot (x - \lambda\mu)] = 0. \quad (3.2)$$

Under the new density $g(x)$, one has to show that the above expression evaluated at α' is positive. Consequently, we want to show that

$$\int_0^L U'(Z) (x - \lambda\mu) s(x) dx > 0 \quad (3.3)$$

where $s(x) \equiv g(x) - f(x)$ is positive (negative) whenever the likelihood ratio $\frac{f(x)}{g(x)}$ is less (greater) than 1.

In order to establish (3.3), one has to consider three cases: (i) $x_5 \leq \lambda\mu \leq x_6$; (ii) $x_5 \geq \lambda\mu$; (iii) $\lambda\mu \geq x_6$. We shall restrict ourselves to cases (i) and (iii) since the proof can be repeated with appropriate modifications for the other case.

For case (i), we can rewrite (3.3) as:⁵

⁵The proof of case (iii) is presented in the Appendix.

$$\begin{aligned}
& \int_0^{x_5} U'(Z) (x - \lambda\mu) s(x) dx + \int_{x_5}^{\lambda\mu} U'(Z) (x - \lambda\mu) s(x) dx \\
& + \int_{\lambda\mu}^{x_6} U'(Z) (x - \lambda\mu) s(x) dx + \int_{x_6}^L U'(Z) (x - \lambda\mu) s(x) dx \geq 0
\end{aligned} \tag{3.4}$$

where Z stands for $Z(\alpha, x)$. From the definition of a relatively strong increase in risk, the integrands in (3.4) alternate in sign starting with a negative, as can be seen from Figure 2 below where $\lambda\mu$ happens to lie in the interval $[x_5, x_6]$.

Because $U'(Z)$ is increasing in x when $\alpha^* < 1$, the left-hand side of (3.4) is greater than

$$U'(Z_5) \left[\int_0^{\lambda\mu} (x - \lambda\mu) s(x) dx \right] + U'(Z_6) \left[\int_{\lambda\mu}^L (x - \lambda\mu) s(x) dx \right] \tag{3.5}$$

where $Z_i = W_0 - x_i + \alpha^* x_i - \lambda\mu\alpha^*$, $i = 5, 6$.

The sum of the two expressions in brackets above is clearly equal to zero. Indeed

$$\int_0^L (x - \lambda\mu) s(x) dx = \int_0^L x s(x) dx - \lambda\mu \int_0^L s(x) dx \tag{3.6}$$

and the two terms on the R.H.S. of (3.6) are zero because of the definition of a mean preserving increase in risk.⁶

To complete the proof, since $U'(Z_5) < U'(Z_6)$, we now show that the first bracket in (3.5) is negative. Integrating by parts, one obtains

⁶Let us notice that the assumption Z_{xx} plays its role here. Indeed, $Z_{xx} = 0$ implies that the expression on the left-hand side of (3.6) can easily be evaluated.

$$\int_0^{\lambda\mu} (x-\lambda\mu) s(x) dx = (x-\lambda\mu) S(x) \Big|_0^{\lambda\mu} - \int_0^{\lambda\mu} S(x) dx = - \int_0^{\lambda\mu} S(x) dx \quad (3.7)$$

which is negative by the definition of a relatively weak increase in risk.

Q.E.D.

Figure (2) gives an example with the following two triangular densities:

$$\begin{aligned} g(x) &= 1/25x & 0 \leq x \leq 5 = \mu \\ &= -1/25x + 2/5 & \mu = 5 \leq x \leq 10 = L \\ f(x) &= 1/9x - 2/9 & x_2 = 2 \leq x \leq 5 = \mu \\ &= -1/9x + 8/9 & \mu = 5 \leq x \leq 8 = x_3 \end{aligned}$$

(Figure 2, here)

Since $s(x) \equiv g(x) - f(x)$, $s(x)$ is negative in the interval $[x_5, x_6]$ and positive otherwise, which is in accordance with condition (b) in the definition of a relatively strong increase in risk (1.4).

Remark : It is interesting to notice that when $\lambda\mu$ lies in the interval $[x_5, x_6]$ neither condition (c) nor condition (d) in (1.4) are necessary to prove the desired result. In the Appendix we show explicitly how condition (d) is useful for the case $x_6 \leq \lambda\mu$ and the reader can verify that condition (c) has a symmetric role for the case $x_5 \geq \lambda\mu$. Finally, since the Meyer and Ormiston's definition of increasing risk implies that $x_5 = x_2$ and $x_6 = x_3$, $\lambda\mu$ lies, by definition, in the interval $[x_2, x_3]$.

After having made the proof for marginal changes in risk, a few words about a global increase in risk are in order. If $F(x)$ were a degenerate distribution with probability 1 at x_0 inside $[0, L]$, it is clear that a risk averse individual would never buy non-actuarial insurance so that a corner solution ($\alpha^* = 0$) prevails. The transition from certainty to uncertainty (with distribution $G(x)$) will necessarily create a positive insurance demand for a risk averse agent

if λ is not prohibitive ($\lambda < \bar{\lambda}$). Hence, Theorem 3 also applies for a global increase in risk despite the fact that $Z_{\alpha\alpha}$ equals zero.

Finally, we can apply directly the definition of deterministic transformations of random variables to the coinsurance problem and obtain the result that a risk averse individual with decreasing absolute risk aversion will increase α from α' to α'' if a simple increase in risk is applied to the initial random variable. This result can easily be verified since the characteristics of the payoff function in the coinsurance problem (given in Table 1) satisfy the conditions of Theorem 2. (Let us remind you that Condition 2 in Table 1 is unimportant to get the desired results).

CONCLUSION

In the same manner as restrictions on the shift from F to G are substitutes to restrictions on U to obtain intuitively comparative statics results, this paper has shown that restricting the class of the payoff functions to a linear one did not preclude the applicability of well known theorems to the demand for insurance.

This general conclusion, applied here to the coinsurance problem, can of course be extended to other problems with a linear payoff such as the simple portfolio problem or the competitive firm under uncertainty with constant marginal costs [see Dionne and Eeckhoudt (1990)]. Another potential extension concerns other forms of insurance contracts. An example is the choice of the optimal deductible which involves kinked linear payoffs.

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APPENDIX

Proof for the case where $\lambda\mu \geq x_s$

We have to show that

$$\int_0^L U'(Z) (x - \lambda\mu) s(x) dx > 0. \quad (\text{A.1})$$

Let us first consider the case where

$$\int_{x_s}^L (x - \lambda\mu) s(x) dx > 0.$$

By using (3.6) in the text, this implies that

$$\int_0^{x_s} (x - \lambda\mu) s(x) dx < 0$$

and the proof in the text gives the desired result without any additional condition. This is true since (A.1) is greater than the following positive expression:

$$U'(Z_s) \left[\int_0^{x_s} (x - \lambda\mu) s(x) dx \right] + U'(Z_{\lambda\mu}) \left[\int_{x_s}^L (x - \lambda\mu) s(x) dx \right].$$

When

$$\int_{x_s}^L (x - \lambda\mu) s(x) dx < 0 \quad (\text{A.2})$$

we have to introduce condition (d) in (1.4) to get the desired result.

To see this, let us first notice that (A.2) implies

$$\int_0^{x_2} (x - \lambda\mu) s(x) dx > 0$$

and, consequently,

$$\int_0^{x_2} U'(Z) (x - \lambda\mu) s(x) dx > 0.$$

Therefore, by definition,

$$\int_0^L U'(Z) (x - \lambda\mu) s(x) dx > \int_{x_2}^L U'(Z) (x - \lambda\mu) s(x) dx. \quad (\text{A.3})$$

To complete the proof we now show that the right hand side of (A.3) is positive. This expression can be rewritten as

$$\int_{x_2}^{x_3} U'(Z) (x - \lambda\mu) s(x) dx + \int_{x_3}^L U'(Z) (x - \lambda\mu) s(x) dx. \quad (\text{A.4})$$

The second term in (1.4) is non negative. The first integral is equal to

$$\int_{x_2}^{x_3} U'(Z) (x - \lambda\mu) (r(x) - 1) f(x) dx$$

where $r(x) \equiv \frac{g(x)}{f(x)}$. Condition (d) in (1.4) implies that

$$\int_{x_1}^{x_2} U'(Z) (x - \lambda\mu) (f(x) - 1) f(x) dx > (r(\lambda\mu) - 1) \int_{x_1}^{x_2} U'(Z) (x - \lambda\mu) f(x) dx .$$

Finally, since by (3.2)

$$\int_{x_1}^{x_2} U'(Z) (x - \lambda\mu) f(x) dx = 0$$

it follows that

$$\int_{x_1}^{x_2} U'(Z) (x - \lambda\mu) f(x) dx > 0$$

which completes the proof since $r(\lambda\mu) > 1$ by definition of a relatively strong increase in risk.

Q.E.D.

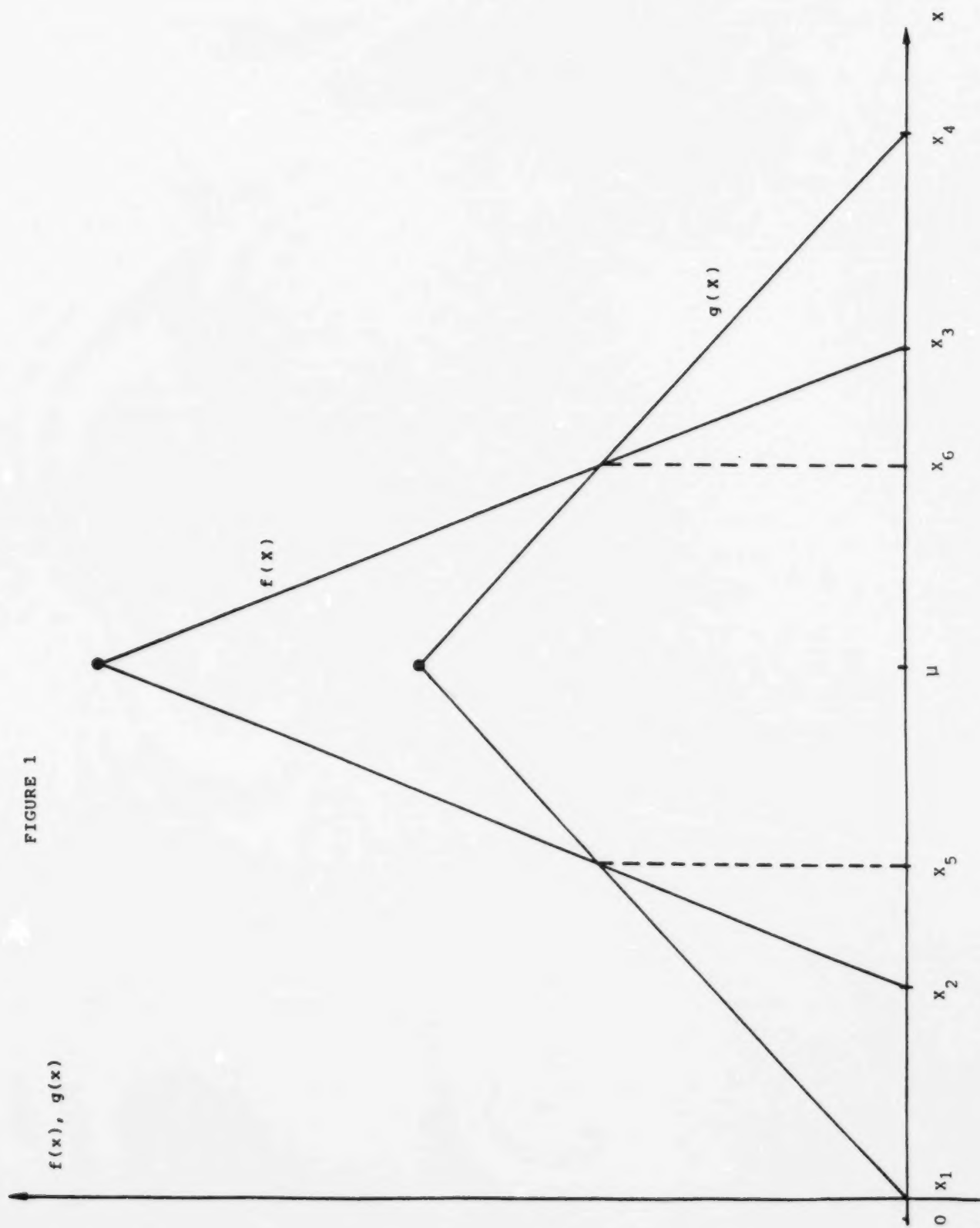
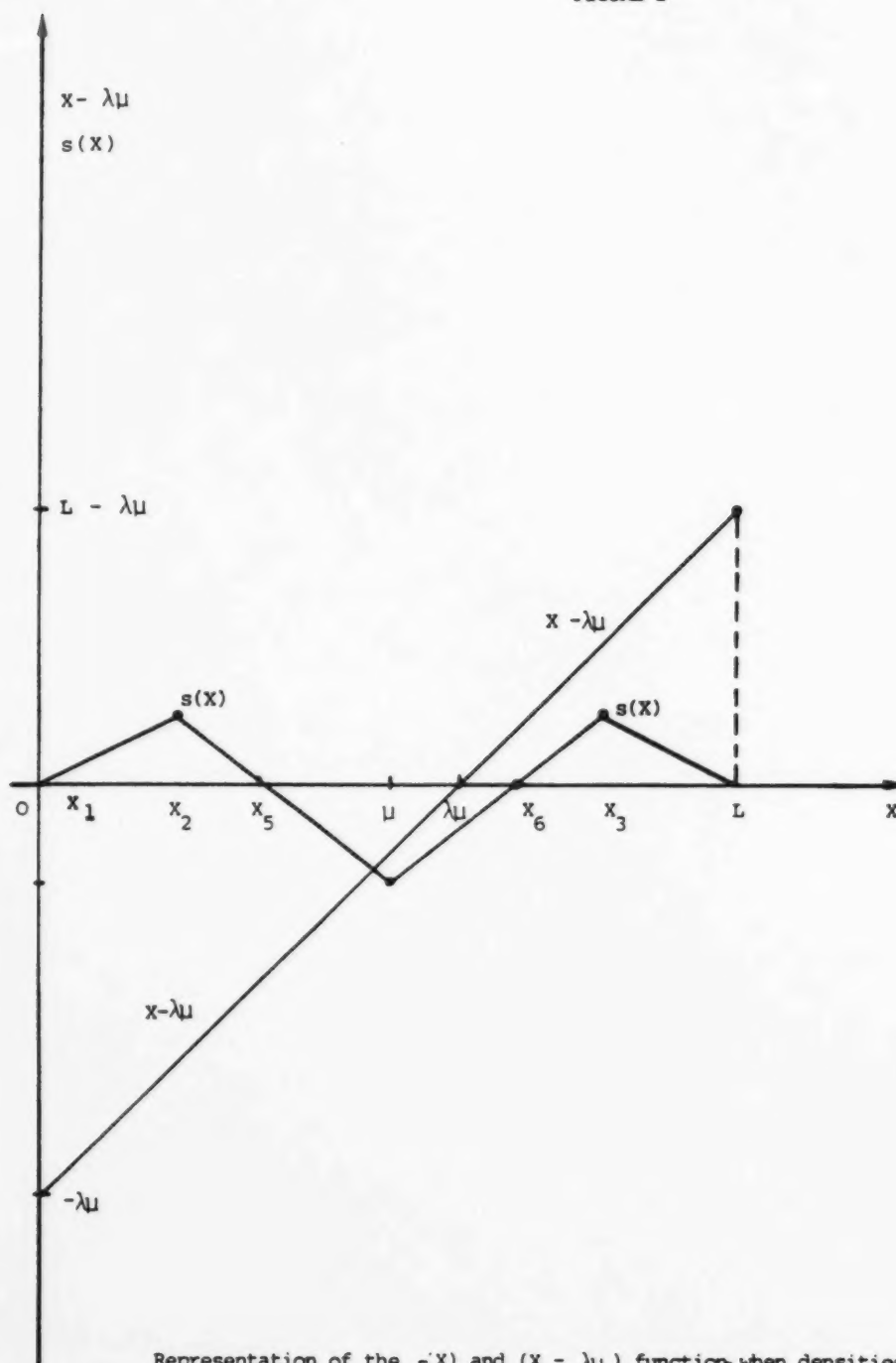


FIGURE 1

An example of a relatively strong increase in risk

FIGURE 2



Representation of the $s(x)$ and $(x - \lambda\mu)$ functions when densities are triangular
and the payoff is linear